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A Supplement on Generalized Pareto Distribution Using Weibull and Dagum distribution

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Abstract—There are many new, enhanced, flexible and robust probability distributions have been produced from the current basic distributions to empower their applications in assorted fields. This paper proposes another distribution called Modified Weibull-Dagum (MWD) distribution. The proposed model is based on the New Weibull–G family (Tahir MH [31]) and its extension of the three parameter Dagum distribution. Various structural properties are derived including explicit expressions for the quantile function, moments. The paper proposes the method of maximum likelihood estimation is used for estimating the parameters of the model. Finally, The applicability of the new model illustrated with two real data sets and its goodness-of-fit compared with that of the different generalization of Pareto distribution by using AIC, AICC BIC, K-Sgoodness-of-fit measures and the results shows the performance of the new distribution better over other generalizations of Pareto distribution.

Keywords — Weibull distribution, Dagum distribution, Weibull-Dagum distribution, Pareto distribution, Reliability, Maximum likelihood estimation

1. INTRODUCTION

The precision and trustworthiness of results of any statistical modelling depend intensely on the decision of model utilized. Right data with wrong model regularly yield ambiguous and misleading estimates and forecasts which leads to incorrect and entangled interpretations and conclusions. This paper presents a new model called the Modified Weibull-Dagum (MWD) distribution. This distribution is an important generalization of the Dagum distribution whose beauty stems from handiness in modeling variables which means, in real world circumstances the distribution have a better fitting capacity in comparison with some other well-known generalized Pareto distributions. Different generalizations of Pareto distribution are studied and helpful for modeling and predicting in a wide assortment of financial settings, there is a positive preferred standpoint in focusing discussion on one explicit field of application: the size distribution of income. Wide assortments of socioeconomic variables have distributions that are heavy-tailed furthermore, sensibly very much fitted by Pareto distributions. So MWD distribution can be used to model the standardized price returns on individual stocks, sizes of sand particles and large casualty losses for certain lines of business, the sizes of human settlement, the values of oil reserves in oil fields, hard disk drive error rates etc.

Dagum[10] introduced Dagum distribution an alternative to the Pareto and log-normal distribution has been utilized to examine income and wage distribution as well as wealth distribution and its features widely examined for modeling personal income data. Its applications tohuman personal income and capital appeared in Costa [26], Ivana [23], Perez and Alaiz [12], Lukasiewicz et al [34], Matejka and Duspivova [28], Kleiber and Kotz [13], Kleiber [11], Shehzad and Asghar [32] and Pant and Headrick [29] studied about properties and parameter estimation of the Dagum distribution.

A random variable X has the Dagum distribution with parameters a,b and p , if its pdf and cdf is of the form

$$f(x) = a\,b\,p\,x^{-p-1}\left(1+b\,x_{-p}\right)^{-a-1},\ x>0, a,b,\ p>0 \qquad (1)$$

$$F(x) = \left(1+b\,x^{-p}\right)^{-a},\ x>0, a,b,\ p>0 \qquad (2)$$

Further Dagum distribution have the flexibility of modeling
lifetime data and different generalizations of the distribution have been proposed in literature For example: Log-Dagum distribution [16], Beta Dagum distribution [15], Mc-Dagum distribution [8], Exponentiated Kumaraswamy-Dagum distribution [21], Gamma-Dagum distribution [5], weighted Dagum distribution [7], extended Dagum distribution [2], transmuted Dagum distribution [22], Dagum-Poisson distribution [6], A power log-Dagum distribution: estimation and applications [20] and Odd Log-Logistic Dagum Distribution: Properties and Applications [18].

The rest of this paper contains the following sections: Section 2 is the introduction of the modified Weibul-Dagum distribution (MWD) Section 3 is the mathematical Properties; Section 4 is the application of the new distribution and Section 5 is the conclusion.

2. MODIFIED WEIBULL DAGUM DISTRIBUTION

This paper is interested in extending the Dagum distribution using the new Weibull–G family of distribution proposed by Tahir et al [31]. The pdf and cdf of the family of distributions given are respectively.

$$f(x) = \alpha\beta \frac{g(x)}{G(x)} \{-\log[G(x)]\}^{\beta-1} e^{-\alpha\{-\log[G(x)]\}^{\beta}} \qquad (3)$$

$$F(x) = e^{-\alpha\{-\log[G(x)]\}^{\beta}} \qquad (4)$$

where $x > 0, \alpha, \beta > 0$ are two positive shape parameters and $g(x)$ and $G(x)$ are the cdf and pdf of any continuous distribution. Here we used Dagum distribution and got the modified Weibull Dagum distribution as given below:

$$f(x) = \frac{\alpha\beta abp \{a \log(1+bx^{-p})\}^{\beta-1} e^{-\alpha\{a\log(1+bx^{-p})\}^{\beta}}}{x^{p+1}(1+bx^{-p})}$$

$$x > 0,\ \alpha, \beta, a, b, p > 0 \qquad (5)$$

$$F(x) = e^{-\alpha\{a\log(1+bx^{-p})\}^{\beta}} \qquad (6)$$

The shapes of the proposed distribution are given below for

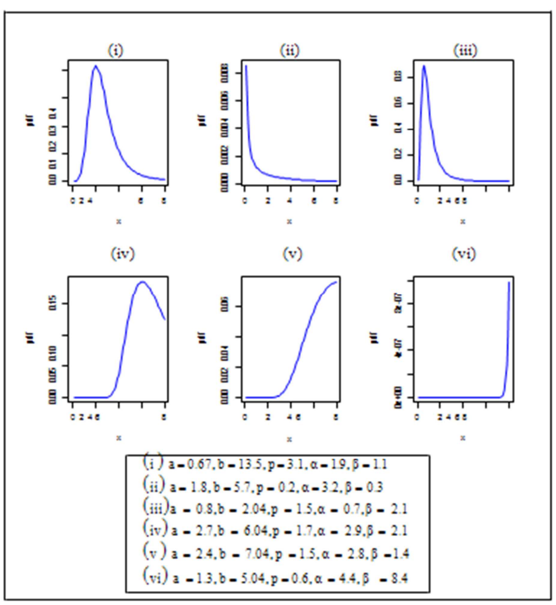

Figure 1

A) Alternative representation of pdf and cdf

Using Power series expansion

$$F(x) = e^{-\alpha\{a\log(1+bx^{-p})\}^{\beta}} = \sum_{i=0}^{\infty} \frac{(-1)^i \alpha^i}{i!} \left\{-\log(1+bx^{-p})^{-a}\right\}^{i\beta}$$

Now, considering the following formula from Tahir et al [31] which holds for $i \geq 1$,

$$\{-\log G(x)\}^{i\beta} = \sum_{k,l=0}^{\infty}\sum_{j=0}^{k} \frac{(-1)^{j+k+l}\, i\beta}{(i\beta-j)} \binom{k-i\beta}{k}\binom{k}{j}\binom{i\beta+k}{l} p_{j,k}\, G(x)^{l} \qquad (8)$$

Where (for $j \geq 0$), $p_{j,0} = 1$ & for ($k = 1,2,3...$)

$$p_{j,k} = k^{-1}\sum_{m=1}^{k} \frac{(-1)^m [m(j+1)-k]}{(m+1)} p_{j,k-m}$$

Therefore, $F(x)$ becomes

$$F(x) = \sum_{i=0}^{\infty}\sum_{k,l=0}^{\infty}\sum_{j=0}^{k} \frac{(-1)^{i+j+k+l}\alpha^i \beta}{(i-1)!(i\beta-j)} \binom{k-i\beta}{k}\binom{k}{j}\binom{i\beta+k}{l} p_{j,k} (1+bx^{-p})^{-al} \qquad (9)$$

By Similar Substitution $f(x)$ becomes:

$$f(x) = \frac{\alpha\beta abp}{x^{p+1}(1+bx^{-p})^{al+1}} \sum_{i=0}^{\infty}\sum_{k,l=0}^{\infty}\sum_{j=0}^{k} \frac{(-1)^{i+j+k+l}\alpha^i}{i!((i+1)\beta-1-j)} \times((i+1)\beta-1)\binom{k-(i+1)\beta+1}{k}\binom{k}{j}\binom{(i+1)\beta-1+k}{l} p_{j,k} \qquad (10)$$

3. MATHEMATICAL PROPERTIES

This section gives some important statistical and mathematical properties of the MWD distribution such as hazard function, moments and moment generating function, quantile function, order Statistics, distribution of minimum, maximum, joint distribution of i^{th} and j^{th} order Statistics, Parameter estimation.

A. Hazard rate function

The investigation of hazard functions emerges normally in lifetime data analysis, a key point of enthusiasm in reliability and biomedical investigations. The following figure shows that the proposed distribution could have increasing, decreasing, and unimodal hazard rate functions A Lifetime distribution with unimodal hazard shape can be utilized to model circumstances like survivability after medical surgery, where the risk rapidly increases because of the chances of complications, such as infection and after that decreases as the patient recovers. The hazard rate function is given by

$$h(x) = \frac{\alpha\beta abp\{a\log\delta\}^{\beta-1} e^{-\alpha\{a\log\delta\}^{\beta}}}{x^{p+1}\,\delta\left(1-e^{-\alpha\{a\log\delta\}^{\beta}}\right)} \qquad (11)$$

Where $\delta = 1 + bx^{-p}$

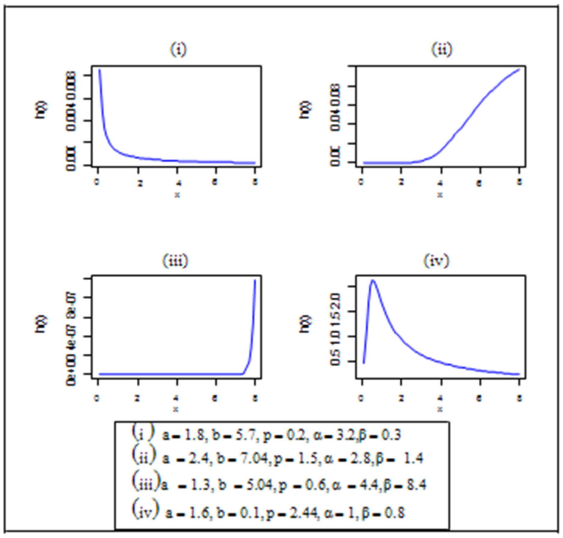

Figure 2

B) Moments and generating function

Moments are the important characteristics of a distribution. This section defines various moments for MWD distribution.

Let $X \sim$ MWD (α, β, a, b, p), for $r = 1,2,3,\ldots$ The r^{th} moment is given by

$$E\left(X^r\right)=\int_0^\infty x^r f(x)dx=\int_0^\infty \frac{\cdots x^{-r}\left(1+bx^{-p}\right)^{-al-1}}{x^{p+1}}dx$$

$$E\left(X^r\right)=\varphi\alpha\beta a b^{rp}B\left(1-\frac{r}{p}, a(l+n)+\frac{r}{p}\right) \quad (12)$$

Where B .,. is the Beta type 2 function defined by ,

$$B(m,n)=\int_0^\infty x^m(1+x)^{-(m+n)}dx \quad m,n>0$$

In particular, the mean of MWD distribution is given by,

$$E(X)=\varphi\alpha\beta a b^{p}B\left(1-\frac{1}{p}, a(l+p)+\frac{1}{p}\right)$$

$$E\left(X^2\right)=\varphi\alpha\beta a b^{2p}B\left(1-\frac{2}{p}, a(l+p)+\frac{2}{p}\right) \quad (14)$$

The moment generating function is given by

$$M_X(t)=E\left(e^{tX}\right)=E\left[\sum_{s=0}^\infty \frac{(tX)^s}{s!}\right]=\sum_{s=0}^\infty \frac{t^s}{s!}E\left(X^s\right)$$

$$M_X(t)=\varphi\alpha\beta a\sum_{s=0}^\infty \frac{t^s}{s!} b^{s/p} B\left(1-\frac{s}{p}, a(l+n)+\frac{s}{p}\right)$$

The characteristic function is

$$\varphi_X(t)=E\left(e^{itX}\right)=E(\cos tX)+iE(\sin tX)=\sum_{k=0}^\infty \frac{(-1)^k t^{2k}}{(2k)!}E\left(X^{2k}\right)+i\sum_{k=0}^\infty \frac{(-1)^k t^{2k+1}}{(2k+1)!}E\left(X^{2k+1}\right)$$

$$=\varphi\alpha\beta a\sum_{k=0}^\infty \frac{(-1)^k t^{2k}}{(2k)!}b^{2kp}B\left(1-2kp, al+2kp\right)+i\sum_{k=0}^\infty \frac{(-1)^k t^{2k+1}}{(2k+1)!}b^{(2k+1)p}\times B\left(1-(2k+1)p, al+(2k+1)p\right) \quad (16)$$

C) Quantile function

The quantile function can obtain by inverting the cumulative distribution of the random variable. It can be used to generate random samples from the proposed distribution.

The q^{th} quantile of the $X_q=F^{-1}(q)=Q(q)$ MWD distribution is given by

$$x_q=\left\{\frac{e^{\left(\frac{-\log u}{\alpha}\right)^{1/\beta}}-1}{b}\right\}^{-\frac{1}{p}} \quad (17)$$

The first quartile, median and the third quartile can be found by applying (17). Specifically, for $p = 0.5$ we have the median of the MWD distribution as follows.

$$x_{0.5}=\left\{\frac{e^{\left(\frac{0.6931}{\alpha}\right)^{1/\beta}}-1}{b}\right\}^{-\frac{1}{p}} \quad (18)$$

The Bowley skewness and Moors kurtosis (based on octiles) can be calculated using the following formulae.

$$SK = \frac{Q\left(\frac{3}{4}\right) + Q\left(\frac{1}{4}\right) - 2Q\left(\frac{1}{2}\right)}{Q\left(\frac{3}{4}\right) - Q\left(\frac{1}{4}\right)}$$

$$KR = \frac{\left[Q\left(\frac{7}{8}\right) - Q\left(\frac{5}{8}\right)\right] + \left[Q\left(\frac{3}{8}\right) - Q\left(\frac{1}{8}\right)\right]}{\left[Q\left(\frac{6}{8}\right) - Q\left(\frac{2}{8}\right)\right]}$$

D) Order Statistics

Let $X_1, X_2, \ldots, X_n$ be random sample from MWD distribution with parameters α, β, a, b, p. Let $X_{(1)}, X_{(2)}, \ldots, X_{(n)}$ be the order statistics from the sample The f(x) and F(x) of the r[th] order statistic say $Y = X_{r,n}$ is given by

$$f_Y(y) = \frac{n!}{(r-1)!(n-r)!} F^{r-1}(y)[1 - F(y)]^{n-r} f(y)$$

$$= \frac{n!}{(r-1)!(n-r)!} \sum_{m=0}^{n-r} \binom{n-r}{m} (-1)^m F^{m+r-1}(y) f(y)$$

$$f(y) = \frac{\alpha\beta a b p}{y^{p+1}\left(1+b y^{-p}\right)^{al+1}} \sum_{m=0}^{n-r} \sum_{i=0}^{\infty} \sum_{k,l=0}^{\infty} \sum_{j=0}^{k} \frac{(-1)^{m+i+j+k+l}}{((\ldots))} \binom{n-r}{m} \binom{k-(i+1)\beta+1}{l} \binom{k}{j} \binom{(i+1)\beta-1+k}{k} p_{j,k}$$

$$F(y) = \sum_{j=r}^{n} \binom{n}{j} [F(y)]^{j} [1-F(y)]^{n-j} = \sum_{j=r}^{n} \sum_{i=0}^{n-j} \binom{n}{j} \binom{n-j}{i} (-1)^{i} F^{i+j}(y)$$

$$F(y) = \sum_{j=r}^{n} \sum_{i=0}^{n-j} \binom{n}{j} \binom{n-j}{i} (-1)^{i} \left\{ e^{-\alpha\left(a\log\left(1+by^{-p}\right)\right)^{\beta}} \right\}^{i+j} \tag{20}$$

E. Distribution of minimum and maximum

Let, X1 , X 2 , ... , X n be independently and identically distributed random variables from MWD distribution with parameters *α, β,a,b, p* then the first and *n* order probability density function are given by

$$g_{1:n}(x) = n\left[1 - F(y,\phi)\right]^{n-1} f(y,\phi)$$

For any real number $b > 0$ and for $-1 < Z < 1$, consider the generalized binomial expansion

$$(1-z)^{-b} = \sum_{j=0}^{\infty} \frac{(-1)^j \Gamma b}{j!\,\Gamma(b-j)} z^j$$

$$g_{1:n}(x) = \frac{\alpha\beta a b p}{x^{p+1}\left(1+bx^{-p}\right)^{al+1}} \sum_{i=0}^{\infty} \sum_{k,l=0}^{\infty} \sum_{j=0}^{k} \frac{(-1)^{i+j+k+l}\alpha^{i}\left((i+1)\beta-1\right)}{i!\left((i+1)\beta-1-i\right)} \times \binom{k-(i+1)\beta+1}{l} \binom{k}{j} \binom{(i+1)\beta-1+k}{k} p_{j,k} \tag{21}$$

$$g_{n:n}(x) = n\left[F\left(x_{(n)}:\phi\right)\right]^{n-1} f\left(x_{(n)},\phi\right)$$

$$= \frac{n\alpha\beta a b p}{x_{(n)}^{p+1}\left(1+bx_{(n)}^{-p}\right)^{al+1}} \sum_{i=0}^{\infty} \sum_{k,l=0}^{\infty} \sum_{j=0}^{k} \frac{(-1)^{i+j+k+l}(n\alpha)^{i}\beta}{i!\left((i+1)\beta-1-j\right)} \times \binom{k-(i+1)\beta-1}{l} \binom{k}{j} p_{j,k} \tag{22}$$

F) Joint distribution of i[th] and j[th] order Statistics

The joint distribution of the *i*[th] and *j*[th] order Statistics from MWD distribution is given by

$$f_{i,j:n}(x_i, x_j) = C\left[F(x_i)\right]^{i-1}\left[F(x_j)-F(x_i)\right]^{j-i-1}\left[1-F\left(x_{(j)}\right)\right]^{n-j} f\left(x_{(i)}\right) \times f\left(x_{(j)}\right)$$

$$= C(\alpha\beta a b p)^2 e^{-\alpha Q_i^{\beta}} \left[e^{-\alpha Q_i^{\beta}} - e^{-\alpha Q_j^{\beta}}\right] \times \left[1-e^{-\alpha Q_j^{\beta}}\right]^{n-j} \frac{\left(Q_i Q_j\right)^{\beta-1}\left(x_i x_j\right)^{-(p+1)}}{\left(1+bx_i^{-p}\right)\left(1+bx_j^{-p}\right)} \tag{23}$$

$$\text{Where, } C = \frac{n!}{(i-1)!(j-i-1)!(n-j)!}$$

$$Q_i = a\log\left(1+bx_i^{-p}\right),\ Q_j = a\log\left(1+bx_j^{-p}\right)$$

G) Parameter Estimation

In this section, we estimate the parameters of MWD using method of maximum likelihood estimation. Let $X_1, X_2, \ldots, X_n$ be a random sample from MWD distribution the vector of model parameters. Then the log likelihood function can be written as

$$\log L = n\log(\alpha\beta a b p) - (p+1)\sum_{i=1}^{n}\log x_i + \sum_{i=1}^{n}\log\left\{-\log\left(1+b x_i^{-p}\right)\right\} + \alpha\beta\sum_{i=1}^{n}\left\{\log_{\alpha}\left(1+b x_i^{-p}\right)^{-a}\right\}^{\beta} - \sum_{i=1}^{n}\log\left(1+b x_i^{-p}\right) \quad (24)$$

$$\frac{\partial \log L}{\partial \alpha} = \frac{n}{\alpha} - \sum_{i=1}^{n}\left\{\alpha\log\left(1+b x_i^{-p}\right)\right\}^{\beta}$$

$$\frac{\partial \log L}{\partial \beta} = \frac{n}{\beta} - \alpha\sum_{i=1}^{n}\left\{a\log\left(1+b x_i^{-p}\right)\right\}^{\beta}\log\left\{a\log\left(1+b x_i^{-p}\right)\right\} + \sum_{i=1}^{n}\log a\log\left(1+b x_i^{-p}\right)$$

$$\frac{\partial \log L}{\partial a} = \frac{n+\beta-1}{a} - \frac{\alpha\beta}{a}\sum_{i=1}^{n}\left\{a\log\left(1+b x_i^{-p}\right)\right\}^{\beta}$$

$$\frac{\partial \log L}{\partial b} = \frac{n}{b} - \alpha\beta\sum_{i=1}^{n}\frac{\left\{a\log\left(1+b x_i^{-p}\right)\right\}^{\beta-1}}{x_i^{p}\left(1+b x_i^{-p}\right)} - \sum_{i=1}^{n}\frac{x_i^{-p}}{\left(1+b x_i^{-p}\right)\log\left(1+b x_i^{-p}\right)} - \sum_{i=1}^{n}\frac{x_i^{-p}}{1+b x_i^{-p}}$$

$$\frac{\partial \log L}{\partial p} = \frac{n}{p} - \alpha\beta\sum_{i=1}^{n}\frac{\left\{a\log\left(1+b x_i^{-p}\right)\right\}^{\beta-1}\left(-\log x_i\right)}{x_i^{p}\left(1+b x_i^{-p}\right)} - \sum_{i=1}^{n}\frac{x_i^{-p}\left(-\log x_i\right)}{\left(1+b x_i^{-p}\right)\log\left(1+b x_i^{-p}\right)} \times \beta - 1\; b - \sum_{i=1}^{n}\log x_i - \sum_{i=1}^{n}\frac{b x_i^{-p}\left(-\log x_i\right)}{1+b x_i^{-p}}$$

Let $\hat{a}, \hat{b}, \hat{a}, \hat{b}, \hat{p}$ be the estimates of the unknown parameters. Using numerical method such as Quasi- Newton algorithm, we can optimize the log-likelihood function which will give the maximum likelihood estimates of parameters. Also large sample approximation can be used to intervals for the parameters. Here we treat the vector of parameters as being approximately multivariate normal distribution.

$$\begin{pmatrix}\hat{\alpha}\\ \vdots\end{pmatrix} \sim N\left[\begin{pmatrix}\alpha\\ \vdots\end{pmatrix}, \begin{pmatrix}\hat{V}_{\alpha\alpha} & \hat{V}_{\alpha\beta} & \hat{V}_{\alpha a} & \hat{V}_{\alpha b} & \hat{V}_{\alpha p}\\ \hat{V}_{\beta\alpha} & \hat{V}_{\beta\beta} & \hat{V}_{\beta a} & \hat{V}_{\beta b} & \hat{V}_{\beta p}\\ \vdots & & & & \\ \hat{V}_{b\alpha} & \hat{V}_{b\beta} & \hat{V}_{ba} & \hat{V}_{bb} & \hat{V}_{bp}\\ \hat{V}_{p\alpha} & \hat{V}_{p\beta} & \hat{V}_{pa} & \hat{V}_{pb} & \hat{V}_{pp}\end{pmatrix}\right]$$

The approximate Variance Covariance matrix is given by,

$$\begin{bmatrix}\hat{V}_{\alpha\alpha} & \hat{V}_{\alpha\beta} & \hat{V}_{\alpha a} & \hat{V}_{\alpha b} & \hat{V}_{\alpha p}\\ \hat{V}_{\beta\alpha} & \hat{V}_{\beta\beta} & \hat{V}_{\beta a} & \hat{V}_{\beta b} & \hat{V}_{\beta p}\\ \hat{V}_{a\alpha} & \hat{V}_{a\beta} & \hat{V}_{aa} & \hat{V}_{ab} & \hat{V}_{ap}\\ \hat{V}_{b\alpha} & \hat{V}_{b\beta} & \hat{V}_{ba} & \hat{V}_{bb} & \hat{V}_{bp}\\ \hat{V}_{p\alpha} & \hat{V}_{p\beta} & \hat{V}_{pa} & \hat{V}_{pb} & \hat{V}_{pp}\end{bmatrix} = \begin{bmatrix}A_{11} & A_{12} & A_{13} & A_{14} & A_{15}\\ A_{21} & A_{22} & A_{23} & A_{24} & A_{25}\\ A_{31} & A_{32} & A_{33} & A_{34} & A_{35}\\ A_{41} & A_{42} & A_{43} & A_{44} & A_{45}\\ A_{51} & A_{52} & A_{53} & A_{54} & A_{55}\end{bmatrix}^{-1}$$

$$A_{11} = -\frac{\partial^2 \log L}{\partial\alpha^2}, A_{12} = -\frac{\partial^2 \log L}{\partial\alpha\,\partial\beta}, A_{13} = -\frac{\partial^2 \log L}{\partial\alpha\,\partial a}, A_{14} = -\frac{\partial^2 \log L}{\partial\alpha\,\partial b}$$

$$A_{15} = -\frac{\partial^2 \log L}{\partial\alpha\,\partial p}, A_{21} = -\frac{\partial^2 \log L}{\partial\beta\,\partial\alpha}, A_{22} = -\frac{\partial^2 \log L}{\partial\beta^2}, A_{23} = -\frac{\partial^2 \log L}{\partial\beta\,\partial a}$$

$$A_{24} = -\frac{\partial^2 \log L}{\partial\beta\,\partial b}, A_{25} = -\frac{\partial^2 \log L}{\partial\beta\,\partial p}, A_{31} = -\frac{\partial^2 \log L}{\partial a\,\partial\alpha}, A_{32} = -\frac{\partial^2 \log L}{\partial a\,\partial\beta}$$

$$A_{33} = -\frac{\partial^2 \log L}{\partial a^2}, A_{34} = -\frac{\partial^2 \log L}{\partial a\,\partial b}, A_{35} = -\frac{\partial^2 \log L}{\partial a\,\partial p}, A_{41} = -\frac{\partial^2 \log L}{\partial b\,\partial\alpha}$$

$$A_{42} = -\frac{\partial^2 \log L}{\partial b\,\partial\beta}, A_{43} = -\frac{\partial^2 \log L}{\partial b\,\partial a}, A_{44} = -\frac{\partial^2 \log L}{\partial b^2}, A_{45} = -\frac{\partial^2 \log L}{\partial b\,\partial p}$$

$$A_{51} = -\frac{\partial^2 \log L}{\partial p\,\partial\alpha}, A_{52} = -\frac{\partial^2 \log L}{\partial p\,\partial\beta}, A_{53} = -\frac{\partial^2 \log L}{\partial p\,\partial a}, A_{54} = -\frac{\partial^2 \log L}{\partial p\,\partial b}$$

$$A_{55} = -\frac{\partial^2 \log L}{\partial p^2}$$

The appropriate 100(1– γ)% two sided confidence interval for α, β, a, b, p respectively given by

$$\hat{\alpha} \pm Z_{\gamma/2}\sqrt{\hat{V}_{\alpha\alpha}} \quad \hat{\beta} \pm Z_{\gamma/2}\sqrt{\hat{V}_{\beta\beta}} \quad a \pm Z_{\gamma/2}\sqrt{\hat{V}_{aa}}$$

$$b \pm Z_{\gamma/2}\sqrt{\hat{V}_{bb}} \quad p \pm Z_{\gamma/2}\sqrt{\hat{V}_{pp}}$$

4. APPLICATION

In this section, we have fitted the modified Weibull - Dagum distribution to the two real data sets and compare the performance with some generalized Pareto distributions to show the applicability of the newly proposed model.

A. Data set:1

Here we compare MWD distribution with new generalized Pareto [25], exponentiated Pareto-I [38], Transmuted Pareto [17], Pareto ArcTan [14], Pareto positive stable [24] distributions with respect to the data set consist of observation on breaking stress of carbon fibers (in Gba) studied by Nicholas and Padgett [30] and recently analyzed by Jayakumar et.al [23] using New generalized Pareto distribution. The Data set is given by

3.70, 2.74, 2.73, 2.50, 3.60, 3.11, 3.27 , 2.87, 1.47, 3.11, 4.42, 2.41, 3.19, 3.22, 1.69, 3.28, 3.09, 1.87, 3.15,4.90, 3.75, 2.43, 2.95, 2.97, 3.39, 2.96, 2.53, 2.67, 2.93,3.22, 3.39, 2.81, 4.20, 3.33, 2.55, 3.31, 3.31, 2.85, 2.56, 3.56, 3.15, 2.35, 2.55, 2.59, 2.38, 2.81, 2.77, 2.17, 2.83, 1.92, 1.41, 3.68, 2.97, 1.36, 0.98, 2.76, 4.91, 3.68, 1.84, 1.59, 3.19, 1.57, 0.81, 5.56, 1.73, 1.59, 2.00, 1.22, 1.12, 1.71, 2.17, 1.17, 5.08, 2.48, 1.18, 3.51, 2.17, 1.69, 1.25, 4.38, 1.84, 0.39, 3.68, 2.48, 0.85, 1.61, 2.79, 4.70, 2.03, 1.80, 1.57, 1.08, 2.03, 1.61, 2.12, 1.89, 2.88, 2.82, 2.05, 3.65

B. Data set:2

This second illustration is based on the strengths of 1.5 cm glass fibers, measured at the National Physical Laboratory, England [35]. Many authors are studied and fitted models for the data set. Here we comparing MWD with exponentiated Pareto-I [38] Transmuted Pareto [17], Transmuted exponentiated [1] Pareto Distributions, Classical Pareto. The Data set is given below

0.55, 0.93, 1.25, 1.36, 1.49, 1.52, 1.58, 1.61, 1.64, 1.68, 1.73,1.81, 2 ,0.74, 1.04, 1.27, 1.39, 1.49, 1.53, 1.59, 1.61, 1.66,1.68, 1.76, 1.82, 2.01, 0.77,1.11, 1.28, 1.42, 1.5, 1.54, 1.6,1.62, 1.66, 1.69, 1.76, 1.84, 2.24, 0.81, 1.13, 1.29, 1.48, 1.5, 1.55, 1.61, 1.62, 1.66, 1.7, 1.77, 1.84, 0.84, 1.24, 1.3, 1.48, 1.51, 1.55, 1.61, 1.63, 1.67, 1.7, 1.78, 1.89

TABLE 1: Goodness-of-fit statistics for the Breaking Stress of Carbon Fiber

Model	**Estimates**	**-2**	**AIC**	**BIC**	**AICC**	**K-S**
MWD	$\hat{\alpha}=1.9118$ $\hat{\beta}=1.1413$ $\hat{a}=0.6755$ $\hat{b}=13.5505$ $\hat{p}=3.1106$	298.6	308.6	321.6	310.0	0.1023
NGP	$\hat{\alpha}=1.8848$ $\hat{\beta}=0.0097$ $\hat{\gamma}=0.0016$ $\theta=35.1440$	340.9	348.9	359.4	349.4	0.1702
EP	$\hat{\alpha}=0.4481$ $\hat{k}=1.4770$	360.5	364.5	369.7	364.7	0.2807
TP	$\hat{\alpha}=0.7899$ $\hat{\lambda}=-0.9692$ $\hat{\beta}=0.3900$	439.5	445.5	453.4	445.8	0.3219
PAT	$\hat{\alpha}=0.1283$ $\hat{\lambda}=0.1128$ $\hat{\beta}=0.1349$	478.6	484.6	492.4	484.8	0.4225

TABLE 2: Goodness-of-fit statistics for the Strengths of 1.5 cm Glass Fibers

Model	**Estimates**	$-2l$	**AIC**	**BIC**
MWD	$\hat{\alpha}=5.5093$ $\hat{\beta}=16.2116$ $\hat{a}=7.85937$ $\hat{b}=0.27943$ $\hat{p}=0.84456$	-46.26	56.52	67.244
Transmuted Exponentiated Pareto	$\hat{k}=1.7333$ $\hat{a}=1.4939$ $\hat{\lambda}=-0.9423$	-46.22	98.45	104.88
ponentiated Pareto-I	$\hat{k}=1.7333$ $\hat{a}=1.0451$	-60.21	124.53	124.43
ansmuted Pareto	$\hat{k}=0.5500$ $\hat{a}=1.4589$ $\hat{\lambda}=-0.9492$	-69.86	145.72	152.15
Pareto	$\hat{k}=0.5500$ $\hat{a}=1.0216$	-85.66	175.32	179.61

From the above results, It is observed that the Modified Weibull - Dagum distribution fits better than other generalizations of Pareto distribution in terms of goodness- of-fit measures such as AIC, BIC, AICC and K- S test.

5. CONCLUSION

In this paper, we have been introduced a probability density function with positive support called modified Weibull – Dagum distribution. The results show the performance of this probability distribution might work better (in terms of model fitting) over other generalizations of Pareto distribution. The proposed distribution could have increasing, decreasing, and unimodal hazard rate functions. These highlights make the MWD distribution appropriate for modeling distinctive failure rates that are more likely to be encountered in real life application. In addition, a broad investigation of its mathematical properties has been given, including moments, quantile Reliability functions and some Information theory measures. Further, method of maximum likelihood estimation is used to calculate the estimate of the unknown parameters of the distribution. Finally the new distribution has been applied in two real datasets it has been seen that the proposed distribution provides a very good fit to the data sets.

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